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Encoding orbital angular momentum onto multiple spin states based on a Huffman tree

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Abstract. Based on single-photon spin-orbit entanglement, we propose in this paper an experimental scheme to sort orbital angular momentum (OAM) by cascading conventional polarizing beam splitters. An economical algorithm, taking advantage of the pair's complementary notation of OAM numbers, is designed to separate a mixture of definite OAM. Our scheme provides an alternative technique to encode OAM onto multiple spin states based on a Huffman tree and its potential in optical communication is demonstrated.

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1. Introduction

Allen *et al* [1] recognized that single photons could carry another new degree of freedom of orbital angular momentum (OAM) besides spin angular momentum. As the eigenstates of the quantum spin operator, $|L\rangle$ and $|R\rangle$ (left- and right-handed circular polarization states) carry spin angular momentum of $\pm\hbar$ [2, 3]. Unlike photon spin, OAM is associated with the helical wavefront $\exp(im\phi)$ (m is an integer) and possesses an eigenvalue of $m\hbar$ per photon [1]. Therefore, photon spin can realize one qubit while OAM allows quNits (quantum state in N dimensions) with a single photon [4, 5]. Recently, considerable attention is given to OAM sorting, which is of importance in the fields of quantum information with larger alphabets [6]–[8]. Leach *et al* [9] devised an interferometric technique that can distinguish individual photons in arbitrarily many OAM states and route single photons according to the modulus of individual OAM. They further developed this method to measure the orbital, spin or total angular momenta [10] and succeeded in measuring the sign of OAM of a light beam using a Shack Hartmann wavefront sensor [11]. By exploiting the rotational Doppler shift, the resolution of the OAM spectrum was demonstrated by Vasnetsov *et al* [12]. Also, Zambrini and Barnett [13] proposed an interferometer spanning the angle space to measure the OAM spectrum.

On the other hand, the coupling between spin angular momentum and OAM in an inhomogeneous and anisotropic medium (called q-plates) recently raised a good deal of interest [14]. Based on the q-plates, quantum information transfer between different degrees of freedom [15] and Shannon dimensionality increase [16] were demonstrated. Here, we propose an experimental scheme for sorting OAM by cascading polarizing beam splitters based on an OAM-dependent polarization manipulation. We find that the ability of our scheme to induce spin–orbit coupling provides an alternative technique to encode OAM onto multiple spin states based on a Huffman tree, which could have some potential in optical communication.

2. Sorting OAM by cascading polarizing beam splitters

As depicted in figure 1, the input twisted photons were assumed to be horizontally linearly polarized, namely $|\varphi\rangle_0 = |m\rangle \otimes |H\rangle$, where $|m\rangle$ denotes the OAM eigenstates and the horizontal polarization state $|H\rangle = (|L\rangle + |R\rangle)/\sqrt{2}$, and can be considered as the superposition of left- and right-handed circular polarized components. After passing through the first 45° (fast axis) quarter-wave plate (QWP1), the left- and right-handed circular polarized components become horizontal and vertical ones, respectively, namely

$$\begin{aligned} J_{45^\circ} |L\rangle &\rightarrow (1+i)/\sqrt{2} |H\rangle, \\ J_{45^\circ} |R\rangle &\rightarrow (1-i)/\sqrt{2} |V\rangle, \end{aligned} \tag{1}$$

where $J_{\pm 45^\circ} = \frac{1+i}{2} \begin{bmatrix} 1 & \mp i \\ \mp i & 1 \end{bmatrix}$ are the Jones matrices of $\pm 45^\circ$ quarter-wave plates. Thus they can be separated by a conventional polarizing beam splitter (PBS1) and then travel, respectively, along Alice and Bob's arms. To rotate the phases while maintaining the polarizations of two components in Alice and Bob's arms, here we employ another type of image and phase rotating prisms (P1 and P2) proposed in [10], which are different from the general Dove prisms used in [9]. P1 and P2 are set so that they have a relative angle α and thus impart the horizontal and

vertical components with a phase difference of $\exp(im2\alpha)$. Meanwhile, three internal reflections in P1 or P2 give a 90° phase shift, which is then canceled by an additional quarter-wave plate (QWP2 or QWP3), leaving the polarizations of two components unaffected [10]. So we obtain the following:

$$(1+i)/\sqrt{2}|H\rangle \rightarrow (1+i)/\sqrt{2}|H\rangle \exp(im\alpha), \quad (1-i)/\sqrt{2}|V\rangle \rightarrow (1-i)/\sqrt{2}|V\rangle \exp(-im\alpha). \quad (2)$$

Subsequently, the two components from Alice and Bob recombine in PBS2 and the following -45° QWP4 (J_{-45°) restores the transforms:

$$J_{-45^\circ}|H\rangle \rightarrow \frac{1+i}{\sqrt{2}}|L\rangle, \quad J_{-45^\circ}|V\rangle \rightarrow \frac{-1+i}{\sqrt{2}}|R\rangle. \quad (3)$$

Finally, the output state from PBS2 and QWP4 can be written as

$$|\varphi\rangle_1 \propto \frac{1}{\sqrt{2}} [\exp(im\alpha)|L\rangle + \exp(-im\alpha)|R\rangle] \otimes |m\rangle = \begin{bmatrix} \cos(m\alpha) \\ \sin(m\alpha) \end{bmatrix} \otimes |m\rangle. \quad (4)$$

Obviously, the function of our setup described by equation (4) is very analogous to circular double refraction [17]. This implies that our present device can simulate the optical activity to rotate the polarization, and of special interest is that the rotation angle is proportional to the OAM number. This offers a method of creating an OAM-dependent polarization rotation. For example, if α is set at $\pi/2$, then an even OAM will lead to a horizontal linear polarization, while an odd OAM will lead to a vertical one. Namely, the so-called single-photon spin-orbit entangled state [18, 19], $|\varphi\rangle = a_{m=\text{even}}|H\rangle \otimes |m=\text{even}\rangle + a_{m=\text{odd}}|V\rangle \otimes |m=\text{odd}\rangle$, is created by our present setup. Such an entanglement, therefore, enables the polarization beam splitter (PBS@0), which transmits the photons with horizontal polarization and reflects vertical polarization, to separate the even OAM from the odd ones. This provides us with an effective way to separate two arbitrary OAM states $\langle m_1|$ and $\langle m_2|$, irrespective of whether they are positive or negative. From equation (4), we know that the resultant polarization of the photons with $m_1\hbar$ OAM will be at $\beta_1 = m_1\alpha$, while those with $m_2\hbar$ OAM will be at $\beta_2 = m_2\alpha$. Therefore, the angle between the two polarization directions is $\Delta\beta = (m_1 - m_2)\alpha$, as shown in figure 1(b). If preset $\alpha = \pi/2(m_1 - m_2)$, then the input photons with different OAM of $m_1\hbar$ and $m_2\hbar$ will make their polarizations align orthogonally to each other so that they can be separated directly by a PBS@ β_1 at or by a PBS@0 following two half-wave plates aligned at $\varphi_1 = -\beta_1/2$ (i.e. HWP1@ φ_1) and $\varphi_2 = 0$ (i.e. HWP2@0), respectively.² These angles should be set precisely; otherwise, an OAM state would appear at two exits of the beam splitter and this will affect the cascaded sorting network. It should be noted that in our scheme the light beam does not need to be focused; otherwise, the performance of the prisms may be limited [20].

Let us now outline the scenario of a general sorting network for separating an arbitrary number of OAM. Each basic unit, figure 1(a), can be considered as a binary branching device that divides the input photons into two sets. As shown in figure 1(c), we set P1 and P2 in the first stage so that $\alpha_1 = \pi/2$. As discussed above, the input photons are divided into two sets with even ($m \bmod 2 = 0$) and odd ($m \bmod 2 = 1$) OAM, respectively. In the second stage, we set $\alpha_2 = \pi/4$; then the photons from the first stage with OAM $m \bmod 2 = 0$ (in upper unit) are further divided

² The combination of HWP1@ $-\beta_1/2$ and HWP2@0 implements the polarization rotation: $|\beta_1\rangle \rightarrow |H\rangle$, $|\beta_2\rangle \rightarrow |V\rangle$, and its transformation matrix is $\begin{bmatrix} \cos \beta_1 & \sin \beta_1 \\ -\sin \beta_1 & \cos \beta_1 \end{bmatrix}$.

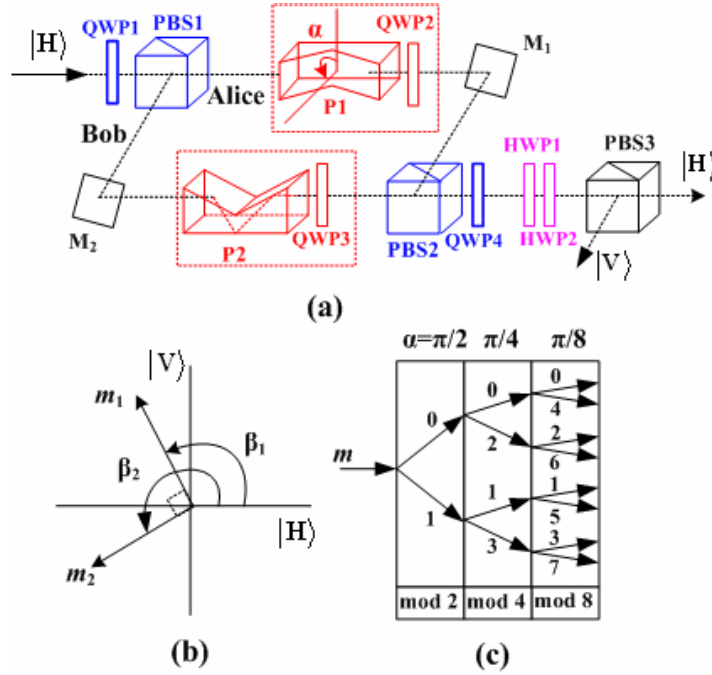


Figure 1. (a) Basic unit for sorting OAM by polarizing beam splitters. QWP, quarter-wave plate; PBS, conventional polarization beam splitter; P, the image and phase rotating prism; M, mirror; HWP, half-wave plate; (b) OAM-dependent polarization rotation and (c) a sorting network for an arbitrary number of OAM.

by PBS@0 into two subsets with $m \bmod 4 = 0$ (corresponding to $\beta_{m=4n} = 0$) and $m \bmod 4 = 2$ (corresponding to $\beta_{m=4n+2} = \pi/2$). The photons with $m \bmod 2 = 1$ (in lower unit) are also divided into two subsets with $m \bmod 4 = 1$ (corresponding to $\beta_{m=4n+1} = \pi/4$) and $m \bmod 4 = 3$ (corresponding to $\beta_{m=4n+3} = 3\pi/4$). For direct separation of OAM $m \bmod 4 = 1$ and $m \bmod 4 = 3$ by a PBS@0, the HWP1 @ -22.5° at the lower unit of the second stage is adjusted to realize the transformation $\pi/4 \rightarrow 0$ and $3\pi/4 \rightarrow \pi/2$. Similarly, in the N th stage with $\alpha_N = \pi/2^N$, the polarizations of photons with $m \bmod 2^N = k$ (corresponding to $\beta_1 = k \cdot \pi/2^N$) and $m \bmod 2^N = 2^{N-1} + k$ (corresponding to $\beta_2 = k \cdot \pi/2^N + \pi/2$) ($k = 0, 1, \dots, 2^{N-1} - 1$) also make an angle $\Delta\beta = \pi/2$, and then can be separated by PBS@0 after the transformation $\beta_1 \rightarrow 0$ and $\beta_2 \rightarrow \pi/2$ by HWP1 @ φ_1 ($\varphi_1 = -\beta_1/2$) and HWP2 @ 0. Along the same line, all photons would be ultimately sorted at specific ports in certain stages, respectively.

One issue is that if the input is a mixture of some definite OAM, some stages or units as shown in figure 1(a) may be redundant. We present here an algorithm for designing an economical sorting network, which takes good advantage of the pair's complementary notation of m [21]. The pair's complement of a binary number is defined as the value obtained by subtracting the number from a large power of two (specifically, from 2^N for an N -bit pair's complement). The bits have a binary radix point and the bits are weighted according to the position of the bit within the array. In the pair's complement, a leading '1' means that the number is negative while a leading '0' means that the number is 0 or positive. One important feature of an N -bit pair's complement notation is that its last j bits give directly the value of m mode 2^j ($j = 1, 2, \dots, N$). For example, for the decimal $m = -4$, its pair's complementary notation

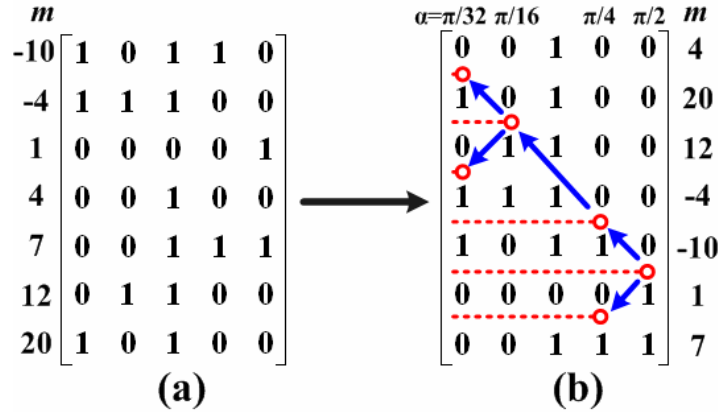


Figure 2. The way of using the pair's complementary notation to construct an economical sorting network.

by a five-bit binary is '11100'. Then we know that $m \bmod 2^1 = 0$, $m \bmod 2^2 = (00)_2 = 0$, $m \bmod 2^3 = (100)_2 = 4$ and $m \bmod 2^4 = (1100)_2 = 12$. Without losing generality, we take a mixture of $\{m = -10, -4, 1, 4, 7, 12, 20\}$ as an example to demonstrate our sorting algorithm. First, we write out all the pair's complementary notations of m and list them in a 7×5 matrix, as shown in figure 2(a). Then, we rearrange this matrix to the form of figure 2(b) according to the rule: scanning the last bit of each row in figure 2(a), and then marking an open circle (red) in the last column to divide the whole matrix into two parts. The rows in the upper part have the same last bit '0' while those in the lower part have '1'. Also, the two open circles in the last second column divide respectively the upper and lower parts into two sub-parts in a similar fashion. Continuing this procedure, finally, every two adjacent rows are separated by an open circle. Let a circle correspond to an aforementioned sorting unit. If the circle lies in the j th last column, then it corresponds to a sorting unit with $\alpha_j = \pi/2^j$ and $\phi_j = -k\pi/2^{j+1}$, where k is the decimal number corresponding to the same last $(j-1)$ bits of two adjacent rows divided by the circle. For example, for the circle in the fourth last column we know that $\alpha_4 = \pi/16$ and $\phi_4 = -\pi/8$, since the same last three bits of two adjacent rows (the second and third rows) are '100', leading to $k = 4$. Based on such configurations, the cascaded binary network is constructed, as illustrated by the red open circles and the blue arrows in figure 2(b). One can see that our sorting network consists of only six sorting units (six red open circles). Comparing with that in [9], our scheme also requires interferometric stability to work. Our scheme, however, does not require any holograms and only conventional half-wave plates are needed to manipulate the desired polarizations. Besides, instead of a non-polarizing beam splitter, we use a polarizing beam splitter to separate orthogonal polarizations. Of interest is that our scheme is completely based on the OAM-dependent polarization manipulation so that it would provide an alternative way to encode OAM onto multiple spin states. In the next section, we will discuss this potential in optical communication.

3. Encoding OAM onto multiple spin states based on a Huffman tree

One of the most promising applications of photonic angular momentum is optical communication. Previous demonstrations used the photon spin to encode information. Two

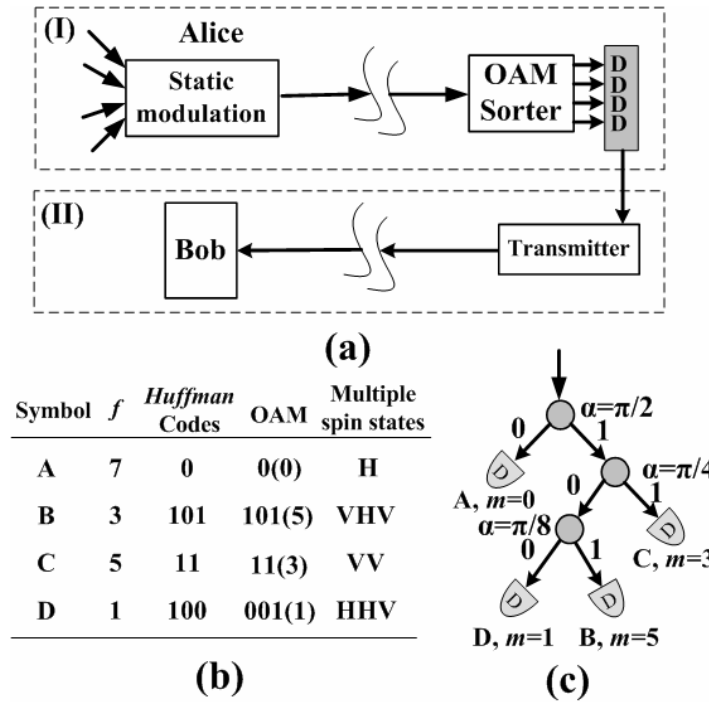


Figure 3. (a) The information link between (I) Alice at a free-space OAM quNits network and (II) Bob at a qubit network; (b) the occurrence frequency f , Huffman codes and OAM codes, encoded multiple spin states for each symbol A, B, C and D, respectively. (c) The OAM encoding and the Huffman codes share the same structure of a binary tree (Huffman tree).

orthogonal spin states are described in a two-dimensional Hilbert space, and thus a photon can take on a logical value of ‘1’ or ‘0’ to act as a qubit. The OAM can, in principle, take on an infinite number of values; therefore, this opens the possibility for realizing the quNits and packing more data into the twisted photons [22, 23]. One issue is that if Alice, at a quNits network, wants to send some data to Bob, at another qubit network, how is their link for the information transfer established? One direct solution is to encode the OAM onto spin states based on our aforementioned sorting network.

Assume that the information Alice wants to send is a sequence of letters ‘ACBACBCBACAADCA’. In the quNits network, Alice can encode this message with OAM by assigning a twist number of a single photon to each letter: $A \rightarrow |m_1\rangle$, $B \rightarrow |m_2\rangle$, $C \rightarrow |m_3\rangle$, and $D \rightarrow |m_4\rangle$. Thus only four different twists are required. Such information coding can be realized by adopting the scheme proposed by [24], as shown in figure 3(a). In this case, sending of different codes is achieved by the switching of an array of angularly separated light sources illuminating the static phase mask. After the static modulation, the input planar photons are converted into twisted ones with coinciding propagation direction and each twist number assigned is selectively dependent on their inclinations in front of the phase mask. An afocal telescope then transmits the encoded twisted photons into the free-space link. Different from [24], the quNits receiver here comprises a similar telescope, the aforementioned OAM sorter, and an array of photodiode detectors (DDDD) aligned to monitor the output intensity of each port and decode the message. Bob could not see this message, since he is located

at another side of the qubit network. The qubit transmitter should read the message decoded by the quNits receiver and encode them as binary codes (spin), and then transfer the binary information to Bob via the qubit link. The transmitter wishes to encode each letter into a set of binary codes, e.g. 0s and 1s, so that no code for a letter is the prefix of the code for any other letter. This prefix property allows us to decode a string of binary codes by repeatedly detecting the prefixes of the string. One technique for finding optimal prefix codes is called Huffman's algorithm [25]. Intuitively speaking, the higher the frequency at which a letter appears, the shorter the set of binary codes. In very long messages containing some infrequent symbols, the saving codes are substantial. In present case, the letter sequence message 'ACBACABCBAACAADCA' consists of four symbols $\{A, B, C, D\}$ and, each occurrence frequency (f) is shown in figure 3(b). When adopting the Huffman algorithm, the four symbols are assigned as $A \rightarrow 0$, $B \rightarrow 101$, $C \rightarrow 11$ and $D \rightarrow 100$ based on the Huffman tree built. The process of building a Huffman tree can be found in [26]. In a Huffman tree, one can think of the binary prefix code being its paths and each leaf representing a letter, see figures 3(b) and (c). At the quNits receiver the scheme for encoding OAM $|m_1\rangle$, $|m_2\rangle$, $|m_3\rangle$ and $|m_4\rangle$ onto spin states should be designed to transmit the information. We find that the structure of the Huffman tree is also particularly suitable for constructing our OAM sorting network, and the OAM encoding can be conveniently performed as follows: $A \rightarrow m_1 = 0(H)$, $B \rightarrow m_2 = (101)_2 = 5(VHV)$, $C \rightarrow m_3 = (11)_2 = 3(VV)$ and $D \rightarrow m_4 = (001)_2 = 1(HHV)$, which are just the reversion of each Huffman codes, where the binary bits '0' and '1' correspond to the horizontal (H) and vertical (V) polarizations, respectively. In the qubit network, each binary code (0 or 1) is represented by a single photon's polarization (H or V). So, based on the Huffman tree, we will be able to economically send the message of a letter sequence with the least number of photons. In each sorting unit, the prism settings are also indicated in figure 3(c). Now we have encoded OAM onto multiple spin states based on a Huffman tree and therefore established the link for information transfer from a quNits node to another qubit node.

4. Conclusion

In summary, we have demonstrated the method of sorting OAM by cascading polarizing beam splitters. An economical algorithm for separating a mixture of definite OAM is devised. We demonstrate that our scheme provides an alternative technique to encode OAM onto spin states based on a Huffman tree and its potential in optical communication is discussed. We expect that our scheme using a single photon's spin degree to sort OAM degree of the same photon would have potential in the spin-orbit hyperentanglement analysis [27].

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